# Lightlike Braneworlds in Anti-de Sitter Bulk Space-times

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**Abstract** We consider five-dimensional Einstein-Maxwell-Kalb-Ramond system self-consistently coupled to a *lightlike* 3-brane, where the latter acts as material, charge and variable cosmological constant source. We find wormhole-like solutions whose total space-time manifold consists of either (a) two "universes", which are identical copies of the exterior space-time region (beyond the horizon) of 5-dimensional Schwarzschild-anti-de Sitter black hole, or (b) a "right" "universe" comprising the exterior space-time region of Reissner-Nordström-anti-de Sitter black hole and a "left" "universe" being the Rindler "wedge" of 5-dimensional flat Minkowski space. The wormhole "throat" connecting these "universes", which is located on their common horizons, is self-consistently occupied by the lightlike 3-brane as a direct result of its dynamics given by an explicit reparametrization-invariant world-volume Lagrangian action. The intrinsic world-volume metric on the 3-brane turns out to be flat, which allows its interpretation as a *lightlike* braneworld.

## 1 Introduction

Lightlike branes (*"LL-branes"* for short) play an important role in modern general relativity. *LL-branes* are singular null (lightlike) hypersurfaces in Riemannian space-time which provide dynamical description of various physically important phenomena in cosmology and astrophysics such as: (i) impulsive lightlike signals arising in cataclysmic astrophysical events (supernovae, neutron star collisions)

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[1]; (ii) dynamics of horizons in black hole physics – the so called "membrane paradigm" [2]; (iii) the thin-wall approach to domain walls coupled to gravity [3]–[6]. More recently, *LL-branes* became significant also in the context of modern non-perturbative string theory [7, 8, 9, 10]

In our previous papers [11]–[20] we have provided an explicit reparametrization invariant world-volume Lagrangian formulation of lightlike *p*-branes (a brief review is given in Section 2) and we have used them to construct various types of wormhole, regular black hole and lightlike braneworld solutions in $D=4$ or higher-dimensional asymptotically flat or asymptotically anti-de Sitter bulk space-times (for a detailed account of the general theory of wormholes see the book [21] and also refs.[22]–[28]). In particular, in refs.[18, 19, 20] we have shown that lightlike branes can trigger a series of spontaneous compactification-decompactification transitions of space-time regions, *e.g.*, from ordinary compactified ("tube-like") Levi-Civita-Bertotti-Robinson [29, 30, 31] space to non-compact Reissner-Nordström or Reissner-Nordström-de-Sitter region or *vice versa*. Let us note that wormholes with "tube-like" structure (and regular black holes with "tube-like" core) have been previously obtained within different contexts in refs.[32]–[40].

Let us emphasize the following characteristic features of *LL-branes* which drastically distinguish them from ordinary Nambu-Goto branes:

(i) They describe intrinsically lightlike modes, whereas Nambu-Goto branes describe massive ones.

(ii) The tension of the *LL-brane* arises as an *additional dynamical degree of freedom*, whereas Nambu-Goto brane tension is a given *ad hoc* constant. The latter characteristic feature significantly distinguishes our *LL-brane* models from the previously proposed *tensionless p*-branes (for a review of the latter, see Ref. [41]) which rather resemble a *p*-dimensional continuous distribution of massless point-particles.

(iii) Consistency of *LL-brane* dynamics in a spherically or axially symmetric gravitational background of codimension one requires the presence of a horizon which is automatically occupied by the *LL-brane* ("horizon straddling" according to the terminology of ref. [4]).

(iv) When the *LL-brane* moves as a *test* brane in spherically or axially symmetric gravitational backgrounds its dynamical tension exhibits exponential "inflation/deflation" time behavior [42] – an effect similar to the "mass inflation" effect around black hole horizons [43, 44].

Here we will focus on studying 4-dimensional lightlike braneworlds in 5-dimensional bulk anti-de Sitter spaces – an alternative to the standard Randall-Sundrum scenario [45, 46] (for a systematic overview to braneworld theory, see [47, 48, 49]). Namely, we will present explicit solutions of 5-dimensionals Einstein-Maxwell-Kalb-Ramond system self-consistently interacting with codimension-one *LL-branes*, which are special kinds of *"wormhole"-like* space-times of either one of the following structures:

(A) two "universes" which are identical copies of the exterior space-time region (beyond the horizon) of 5-dimensional Schwarzschild-anti-de Sitter black hole;

(B) "right" "universe" comprising the exterior space-time region of Reissner-Nordström-anti-de Sitter black hole and "left" "universe" being the Rindler "wedge" of 5-dimensional flat Minkowski space.

Both "right" and "left" "universes" in (A)-(B) are glued together along their common horizons occupied by the *LL-brane* with *flat* 4-dimensional intrinsic world-volume metric, in other words, a flat lightlike braneworld (*LL-braneworld*) at the wormhole "throat". In case (A) the *LL-brane* is electrically neutral whereas in case (B) it is both electrically charged as well as it couples also to a bulk Kalb-Ramond tensor gauge field.

## 2 Lagrangian Formulation of Lightlike Brane Dynamics

In what follows we will consider gravity/gauge-field system self-consistently interacting with a lightlike $p$-brane of codimension one ($D = (p+1)+1$). In a series of previous papers [11]–[20] we have proposed manifestly reparametrization invariant world-volume Lagrangian formulation in several dynamically equivalent forms of *LL-branes* coupled to bulk gravity $G_{\mu\nu}$ and bulk gauge fields, in particular, Maxwell $A_\mu$ and Kalb-Ramond $A_{\mu_1\ldots\mu_{D-1}}$. Here we will use our Polyakov-type formulation given by the world-volume action:

$$S_{\mathrm{LL}}[q,\beta] = -\frac{1}{2}\int d^{p+1}\sigma \, T b_0^{\frac{p-1}{2}} \sqrt{-\gamma}\left[\gamma^{ab}\bar{g}_{ab} - b_0(p-1)\right] , \tag{1}$$

$$-\frac{\beta}{(p+1)!}\int d^{p+1}\sigma \, \varepsilon^{a_1\ldots a_{p+1}}\partial_{a_1}X^{\mu_1}\ldots\partial_{a_{p+1}}X^{\mu_{p+1}}A_{\mu_1\ldots\mu_{p+1}} \tag{2}$$

where:

$$\bar{g}_{ab} \equiv \partial_a X^\mu G_{\mu\nu}\partial_b X^\nu - \frac{1}{T^2}(\partial_a u + qA_a)(\partial_b u + qA_b) \quad , \quad A_a \equiv \partial_a X^\mu A_\mu . \tag{3}$$

Here and below the following notations are used:

- $X^\mu(\sigma)$ are the $p$-brane embedding coordinates in the bulk $D$-dimensional space-time with Riemannian metric $G_{\mu\nu}(x)$ ($\mu,\nu = 0,1,\ldots,D-1$); $(\sigma) \equiv (\sigma^0 \equiv \tau, \sigma^i)$ with $i = 1,\ldots,p$; $\partial_a \equiv \frac{\partial}{\partial\sigma^a}$.
- $\gamma_{ab}$ is the *intrinsic* Riemannian metric on the world-volume with $\gamma = \det\|\gamma_{ab}\|$; $g_{ab}$ is the *induced* metric on the world-volume:

$$g_{ab} \equiv \partial_a X^\mu G_{\mu\nu}(X)\partial_b X^\nu , \tag{4}$$

which becomes *singular* on-shell (manifestation of the lightlike nature), *cf.* Eq.(9) below; $b_0$ is a positive constant measuring the world-volume "cosmological constant".

- $u$ is auxiliary world-volume scalar field defining the lightlike direction of the induced metric (see Eq.(9) below) and it is a non-propagating degree of freedom (cf. ref.[20]).
- $T$ is *dynamical (variable)* brane tension (also a non-propagating degree of freedom).
- The coupling parameters $q$ and $\beta$ are the electric surface charge density and the Kalb-Rammond charge of the *LL-brane*, respectively.

The corresponding equations of motion w.r.t. $X^\mu$, $u$, $\gamma_{ab}$ and $T$ read accordingly (using short-hand notation (3)):

$$\partial_a \left( T \sqrt{|\bar{g}|} \bar{g}^{ab} \partial_b X^\mu \right) + T \sqrt{|\bar{g}|} \bar{g}^{ab} \partial_a X^\lambda \partial_b X^\nu \Gamma^\mu_{\lambda\nu}$$

$$+ \frac{q}{T} \sqrt{|\bar{g}|} \bar{g}^{ab} \partial_a X^\nu (\partial_b u + q A_b) F_{\lambda\nu} G^{\mu\lambda}$$

$$- \frac{\beta}{(p+1)!} \varepsilon^{a_1 \dots a_{p+1}} \partial_{a_1} X^{\mu_1} \dots \partial_{a_{p+1}} X^{\mu_{p+1}} F_{\lambda\mu_1\dots\mu_{p+1}} G^{\lambda\mu} = 0 , \tag{5}$$

$$\partial_a \left( \frac{1}{T} \sqrt{|\bar{g}|} \bar{g}^{ab} (\partial_b u + q A_b) \right) = 0 \quad , \quad \gamma_{ab} = \frac{1}{b_0} \bar{g}_{ab} , \tag{6}$$

$$T^2 + \varepsilon \bar{g}^{ab} (\partial_a u + q A_a)(\partial_b u + q A_b) = 0 . \tag{7}$$

Here $\bar{g} = \det \|\bar{g}_{ab}\|$, $\Gamma^\mu_{\lambda\nu}$ denotes the Christoffel connection for the bulk metric $G_{\mu\nu}$ and:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad , \quad F_{\mu_1\dots\mu_D} = D \partial_{[\mu_1} A_{\mu_2\dots\mu_D]} = \mathscr{F} \sqrt{-G} \varepsilon_{\mu_1\dots\mu_D} \tag{8}$$

are the corresponding gauge field strengths.

The on-shell singularity of the induced metric $g_{ab}$ (4), *i.e.*, the lightlike property, directly follows Eq.(7) and the definition of $\bar{g}_{ab}$ (3):

$$g_{ab} \left( \bar{g}^{bc} (\partial_c u + q A_c) \right) = 0 . \tag{9}$$

Explicit world-volume reparametrization invariance of the *LL-brane* action (1) allows to introduce the standard synchronous gauge-fixing conditions for the intrinsic world-volume metric

$$\gamma^{00} = -1 \quad , \quad \gamma^{0i} = 0 \ (i = 1, \dots, p) . \tag{10}$$

which reduces Eqs.(6)–(7) to the following relations:

$$\frac{(\partial_0 u + q A_0)^2}{T^2} = b_0 + g_{00} \quad , \quad \partial_i u + q A_i = (\partial_0 u + q A_0) g_{0i} (b_0 + g_{00})^{-1} ,$$

$$g_{00} = g^{ij} g_{0i} g_{0j} \ , \ \partial_0 \left( \sqrt{g^{(p)}} \right) + \partial_i \left( \sqrt{g^{(p)}} g^{ij} g_{0j} \right) = 0 \ , \ g^{(p)} \equiv \det \|g_{ij}\| , \tag{11}$$

(recall that $g_{00}, g_{0i}, g_{ij}$ are the components of the induced metric (4); $g^{ij}$ is the inverse matrix of $g_{ij}$). Then, as shown in refs.[11]–[20], consistency of *LL-brane* dynamics in static "spherically-symmetric"-type backgrounds (in what follows we will use Eddington-Finkelstein coordinates, $dt = dv - \frac{d\eta}{A(\eta)}$):

$$ds^2 = -A(\eta)dv^2 + 2dvd\eta + C(\eta)h_{ij}(\theta)d\theta^i d\theta^j \,,$$
$$F_{v\eta} = F_{v\eta}(\eta) \,, \text{ rest} = 0 \quad , \quad \mathscr{F} = \mathscr{F}(\eta) \,, \tag{12}$$

with the standard embedding ansatz:

$$X^0 \equiv v = \tau \quad , \quad X^1 \equiv \eta = \eta(\tau) \quad , \quad X^i \equiv \theta^i = \sigma^i \ (i = 1, \ldots, p) \,. \tag{13}$$

requires the corresponding background (12) to possess a horizon at some $\eta = \eta_0$, which is automatically occupied by the *LL-brane*.

Indeed, in the case of (12)–(13) Eqs.(11) reduce to:

$$g_{00} = 0 \ , \ \partial_0 C(\eta(\tau)) \equiv \dot{\eta} \, \partial_\eta C \big|_{\eta=\eta(\tau)} = 0 \ , \ \frac{(\partial_0 u + qA_0)^2}{T^2} = b_0 \ , \ \partial_i u = 0 \tag{14}$$

($\dot{\eta} \equiv \partial_0 \eta \equiv \partial_\tau \eta(\tau)$). Thus, in the generic case of non-trivial dependence of $C(\eta)$ on the "radial-like" coordinate $\eta$, the first two relations in (14) yield:

$$\dot{\eta} = \frac{1}{2}A(\eta(\tau)) \quad , \quad \dot{\eta} = 0 \quad \rightarrow \quad \eta(\tau) = \eta_0 = \text{const} \quad , \quad A(\eta_0) = 0 \,. \tag{15}$$

The latter property is called "horizon straddling" according to the terminology of ref.[4]. Similar "horizon straddling" has been found also for *LL-branes* moving in rotating axially symmetric (Kerr or Kerr-Newman) and rotating cylindrically symmetric black hole backgrounds [16, 17].

## 3 Gravity/Gauge-Field System Interacting with Lightlike Brane

The generally covariant and manifestly world-volume reparametrization-invariant Lagrangian action describing a bulk Einstein-Maxwell-Kalb-Ramond system (with bulk cosmological constant $\Lambda$) self-consistently interacting with a codimension-one *LL-brane* is given by:

$$S = \int d^D x \sqrt{-G} \left[ \frac{R(G) - 2\Lambda}{16\pi} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{D!2}F_{\mu_1\ldots\mu_D}F^{\mu_1\ldots\mu_D} \right] + S_{\text{LL}}[q, \beta] \,, \tag{16}$$

where again $F_{\mu\nu}$ and $F_{\mu_1\ldots\mu_D}$ are the Maxwell and Kalb-Ramond field-strengths (8) and $S_{\text{LL}}[q, \beta]$ indicates the world-volume action of the *LL-brane* of the form (1). It is now the *LL-brane* which will be the material and charge source for gravity

and electromagnetism, as well as it will generate dynamically an additional space-varying bulk cosmological constant (see Eq.(20) and second relation (28) below).

The equations of motion resulting from (16) read:

(a) Einstein equations:

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R + \Lambda G_{\mu\nu} = 8\pi\left(T^{(EM)}_{\mu\nu} + T^{(KR)}_{\mu\nu} + T^{(brane)}_{\mu\nu}\right) ; \qquad (17)$$

(b) Maxwell equations:

$$\partial_\nu\left[\sqrt{-G}F_{\kappa\lambda}G^{\mu\kappa}G^{\nu\lambda}\right] + j^\mu_{(brane)} = 0 ; \qquad (18)$$

(c) Kalb-Ramond equations (recall definition of $\mathcal{F}$ in (8)):

$$\varepsilon^{\nu\mu_1\ldots\mu_{p+1}}\partial_\nu\mathcal{F} - J^{\mu_1\ldots\mu_{p+1}}_{(brane)} = 0 ; \qquad (19)$$

(d) The *LL-brane* equations of motion have already been written down in (5)–(7) above.

The energy-momentum tensors of bulk gauge fields are given by:

$$T^{(EM)}_{\mu\nu} = F_{\mu\kappa}F^{\mu\nu} - G_{\mu\nu}\frac{1}{4}F_{\kappa\lambda}F^{\kappa\lambda} \quad , \quad T^{(KR)}_{\mu\nu} = -\frac{1}{2}\mathcal{F}^2 G_{\mu\nu} , \qquad (20)$$

where the last relation indicates that $\Lambda \equiv 4\pi\mathcal{F}^2$ can be interpreted as dynamically generated cosmological "constant".

The energy-momentum (stress-energy) tensor $T^{(brane)}_{\mu\nu}$ and the electromagnetic $j^\mu_{(brane)}$ and Kalb-Ramond $J^{\mu_1\ldots\mu_{p+1}}_{(brane)}$ charge current densities of the *LL-brane* are straightforwardly derived from the pertinent *LL-brane* action (1):

$$T^{\mu\nu}_{(brane)} = -\int d^{p+1}\sigma\,\frac{\delta^{(D)}\left(x - X(\sigma)\right)}{\sqrt{-G}}T\sqrt{|\bar{g}|}\bar{g}^{ab}\partial_a X^\mu \partial_b X^\nu , \quad (21)$$

$$j^\mu_{(brane)} = -q\int d^{p+1}\sigma\,\delta^{(D)}\left(x - X(\sigma)\right)\sqrt{|\bar{g}|}\bar{g}^{ab}\partial_a X^\mu\left(\partial_b u + qA_b\right)T^{-1} , \quad (22)$$

$$J^{\mu_1\ldots\mu_{p+1}}_{(brane)} = \beta\int d^{p+1}\sigma\,\delta^{(D)}\left(x - X(\sigma)\right)\varepsilon^{a_1\ldots a_{p+1}}\partial_{a_1}X^{\mu_1}\ldots\partial_{a_{p+1}}X^{\mu_{p+1}} . \quad (23)$$

Construction of "wormhole"-like solutions of static "spherically-symmetric"-type (12) for the coupled gravity-gauge-field-*LL-brane* system (16) proceeds along the following simple steps:

(i) Choose "vacuum" static "spherically-symmetric"-type solutions (12) of (17)–(19) (*i.e.*, without the delta-function terms due to the *LL-branes*) in each region $-\infty < \eta < \eta_0$ and $\eta_0 < \eta < \infty$ with a common horizon at $\eta = \eta_0$;

(ii) The *LL-brane* automatically locates itself on the horizon according to "horizon straddling" property (15);

(iii) Match the discontinuities of the derivatives of the metric and the gauge field strength (12) across the horizon at $\eta = \eta_0$ using the explicit expressions for the

*LL-brane* stress-energy tensor, electromagnetic and Kalb-Ramond charge current densities (21)–(23).

Using (11)–(13) we find:

$$T^{\mu\nu}_{(\text{brane})} = S^{\mu\nu}\,\delta(\eta - \eta_0) \quad , \quad j^{\mu}_{(\text{brane})} = \delta^{\mu}_0 q\sqrt{\det\|G_{ij}\|}\,\delta(\eta - \eta_0)\,, \qquad (24)$$

$$\frac{1}{(p+1)!}\varepsilon_{\mu\nu_1\ldots\nu_{p+1}}J^{\nu_1\ldots\nu_{p+1}} = \beta\,\delta^{\eta}_{\mu}\,\delta(\eta - \eta_0)$$

where $G_{ij} = C(\eta)h_{ij}(\theta)$ (*cf.* (12)) and the surface energy-momentum tensor reads:

$$S^{\mu\nu} \equiv \frac{T}{b_0^{1/2}}\left(\partial_\tau X^\mu \partial_\tau X^\nu - b_0 G^{ij}\partial_i X^\mu \partial_j X^\nu\right)_{\nu=\tau,\,\eta=\eta_0,\,\theta^i=\sigma^i}\,. \qquad (25)$$

The non-zero components of $S_{\mu\nu}$ (with lower indices) and its trace are:

$$S_{\eta\eta} = \frac{T}{b_0^{1/2}} \quad , \quad S_{ij} = -Tb_0^{1/2}G_{ij} \quad , \quad S^{\lambda}_{\lambda} = -pTb_0^{1/2}\,. \qquad (26)$$

Taking into account (24)–(26) together with (12)–(15), the matching relations at the horizon $\eta = \eta_0$ become [18, 19, 20] (for a systematic introduction to the formalism of matching different bulk space-time geometries on codimension-one hypersurfaces ("thin shells") see the textbook [50]):

(i) Matching relations from Einstein eqs.(17):

$$\left[\partial_\eta A\right]_{\eta_0} = -16\pi T\sqrt{b_0} \quad , \quad \left[\partial_\eta \ln C\right]_{\eta_0} = -\frac{16\pi}{p\sqrt{b_0}}T \qquad (27)$$

with notation $\left[Y\right]_{\eta_0} \equiv Y\big|_{\eta\to\eta_0+0} - Y\big|_{\eta\to\eta_0-0}$ for any quantity $Y$.

(ii) Matching relation from gauge field eqs.(18)–(19):

$$\left[F_{\nu\eta}\right]_{\eta_0} = q \quad , \quad \left[\mathscr{F}\right]_{\eta_0} = -\beta\,. \qquad (28)$$

(iii) $X^0$-equation of motion of the *LL-brane* (the only non-trivial contribution of second-order *LL-brane* eqs.(5) in the case of embedding (13)):

$$\frac{T}{2}\left(\langle\partial_\eta A\rangle_{\eta_0} + pb_0\langle\partial_\eta \ln C\rangle_{\eta_0}\right) - \sqrt{b_0}\left(q\langle F_{\nu\eta}\rangle_{\eta_0} - \beta\langle\mathscr{F}\rangle_{\eta=\eta_0}\right) = 0 \qquad (29)$$

with notation $\langle Y\rangle_{\eta_0} \equiv \frac{1}{2}\left(Y\big|_{\eta\to\eta_0+0} + Y\big|_{\eta\to\eta_0-0}\right)$.

## 4 Explicit Solutions: Braneworlds via Lightlike Brane

Consider 5-dimensional AdS-Schwarzschild black hole in Eddington-Finkelstein coordinates $(v, r, \mathbf{x})$ (with $\mathbf{x} \equiv (x^1, x^2, x^3)$):

$$ds^2 = -A(r)dv^2 + 2dv\,dr + Kr^2 d\mathbf{x}^2 \quad , \quad A(r) = Kr^2 - m/r^2 \,, \tag{30}$$

where $\Lambda = -6K$ is the bare negative 5-dimensional cosmological constant and $m$ is the mass parameter of the black hole. The pertinent horizon is located at:

$$A(r_0) = 0 \rightarrow r_0 = (m/K)^{1/4} \quad , \quad \text{where } \partial_r A(r_0) > 0 \,. \tag{31}$$

First, let us consider self-consistent Einstein-*LL-brane* system (16) with a neutral *LL-brane* source (*i.e.* no *LL-brane* couplings to bulk Maxwell and Kalb-Ramond gauge fields: $q, \beta = 0$ in $S_{\mathrm{LL}}[q, \beta]$). A simple trick to obtain *"wormhole"*-like solution to this coupled system is to change variables in (30):

$$r \rightarrow r(\eta) = r_0 + |\eta| \tag{32}$$

with $r_0$ being the AdS-Schwarzschild horizon (31), where now $\eta \in (-\infty, +\infty)$, *i.e.*, consider:

$$ds^2 = -A(\eta)dv^2 + 2dv\,d\eta + C(\eta)d\mathbf{x}^2 \,, \tag{33}$$

$$A(\eta) = K(r_0 + |\eta|)^2 - \frac{m}{(r_0 + |\eta|)^2} \quad , \quad C(\eta) = K(r_0 + |\eta|)^2 \,, \tag{34}$$

$$A(0) = 0 \quad , \quad A(\eta) > 0 \text{ for } \eta \neq 0 \,.$$

Obviously, (32) is *not* a smooth local coordinate transformation due to $|\eta|$. The coefficients of the new metric (33)–(34) are continuous at the horizon $\eta_0 = 0$ with discontinuous first derivates across the horizon. The *LL-brane* automatically locates itself on the horizon according to the "horizon-straddling" property of its world-volume dynamics (15).

Substituting (33)–(34) into the matching relations (27)–(29) we find the following relation between bulk space-time parameters $(K = |\Lambda|/6, m)$ and the *LL-brane* parameters $(T, b_0)$ :

$$T^2 = \frac{3}{8\pi^2}K \quad , \quad T < 0 \quad , \quad b_0 = \frac{2}{3}\sqrt{Km} \,. \tag{35}$$

Taking into account second Eq.(6) and (10) the intrinsic metric $\gamma_{ab}$ on the *LL-braneworld* becomes *flat*:

$$\gamma_{00} = -1 \quad , \quad \gamma_{0i} = 0 \quad , \quad \gamma_{ij} = \frac{3}{2}\delta_{ij} \,. \tag{36}$$

The solution (34)–(36) describes a "wormhole"-like $D = 5$ bulk space-time consisting of two "universes" being identical copies of the exterior region beyond the

horizon ($r > r_0$) of the 5-dimensional AdS-Schwarzschild black hole glued together along their common horizon (at $r = r_0$) by the *LL-brane*, *i.e.*, the latter serving as a wormhole "throat", which in turn can be viewed as a *LL-braneworld* with flat intrinsic geometry (36).

Let us now consider the 5-dimensional AdS-Reissner-Nordström black hole (in Eddington-Finkelstein coordinates $(v, r, \mathbf{x})$):

$$ds^2 = -A(r)dv^2 + 2dvdr + Kr^2d\mathbf{x}^2 \quad, \quad \Lambda = -6K,$$

$$A(r) = Kr^2 - \frac{m}{r^2} + \frac{Q}{r^4} \quad, \quad F_{vr} = \sqrt{\frac{3}{4\pi}}\frac{Q}{r^3}. \tag{37}$$

We can construct, following the same procedure, another *non-symmetric* "wormhole"-like solution with a flat *LL-braneworld* occupying its "throat" provided the *LL-brane* is electrically charged and couples to bulk Kalb-Ramond gauge field, *i.e.*, $q, \beta \neq 0$ in (16), (1). This solution describes:

(a) "left" universe being a 5-dimensional flat Rindler space-time – the Rindler "wedge" of $D = 5$ Minkowski space [51, 52] (here $|\eta| = X^2$, where $X$ is the standard Rindler coordinate):

$$ds^2 = \eta dv^2 + 2dvd\eta + d\mathbf{x}^2 \quad, \quad \text{for } \eta < 0; \tag{38}$$

(b) "right" universe comprizing the exterior $D = 5$ space-time region of the AdS-Reissner-Nordström black hole beyond the *outer* AdS-Reissner-Nordström horizon $r_0$ ($A(r_0) = 0$ with $A(r)$ as in (37) and where again we apply the non-smooth coordinate change (32)):

$$ds^2 = -A(\eta)dv^2 + 2dvd\eta + K(r_0 + \eta)^2d\mathbf{x}^2 \tag{39}$$

$$A(\eta) = K(r_0 + \eta)^2 - \frac{m}{(r_0 + \eta)^2} + \frac{Q^2}{(r_0 + \eta)^4} \tag{40}$$

$$F_{v\eta} = \sqrt{\frac{3}{4\pi}}\frac{Q}{(r_0 + \eta)^3} \quad, \quad A(0) = 0 \,, \; \partial_\eta A(0) > 0 \,, \quad \text{for } \eta > 0. \tag{41}$$

All physical parameters of the "wormhole"-like solution (38)–(41) are determined in terms of $(q, \beta)$ – the electric and Kalb-Ramond *LL-braneworld* charges:

$$m = \frac{3}{2\pi\beta^2}\left(1 + \frac{2q^2}{\beta^2}\right) \quad, \quad Q^2 = \frac{9q^2}{2\pi\beta^6} \quad, \quad |\Lambda| \equiv 6K = 4\pi\beta^2 \tag{42}$$

$$|T| = \frac{1}{8\pi}\sqrt{\frac{3}{2}\sqrt{K} + 4\pi(\beta^2 - q^2)} \,, \; b_0 = \frac{1}{6\sqrt{K}}\left[1 + \frac{8\pi}{3}\sqrt{K}(\beta^2 - q^2)\right] \tag{43}$$

Here again $T < 0$. Let is stress the importance of the third relation in (42). Namely, the dynamically generated space-varying effective cosmological constant (cf. second Eq.(20)) through the Kalb-Ramond coupling of the *LL-brane* (cf. second matching relation in (28)) has zero value in the "right" AdS-Reissner-Nordström "uni-

verse" and has positive value $4\pi\beta^2$ in the "left" flat Rindler "universe" (38) compensating the negative bare cosmological constant $\Lambda$.

The intrinsic metric $\gamma_{ab}$ on the *LL-braneworld* is again *flat*:

$$\gamma_{00} = -1 \quad , \quad \gamma_{0i} = 0 \quad , \quad \gamma_{ij} = \frac{1}{b_0}\delta_{ij} \tag{44}$$

## 5 Traversability and Trapping Near the Lightlike Braneworld

The "wormhole"-like solutions presented in the previous Section share the following important properties:

(a) The *LL-braneworlds* at the wormhole "throats" represent "exotic" matter with $T < 0$, *i.e.*, negative brane tension implying violation of the null-energy conditions as predicted by general wormhole arguments [21] (although the latter could be remedied via quantum fluctuations).

(b) The wormhole space-times constructed via *LL-branes* at their "throats" are *not* traversable w.r.t. the "laboratory" time of a static observer in either of the different "universes" comprising the pertinent wormhole space-time manifold since the *LL-branes* sitting at the "throats" look as black hole horizons to the static observer. On the other hand, these wormholes *are traversable* w.r.t. the *proper time* of a traveling observer.

Indeed, proper-time traversability can be easily seen by considering dynamics of test particle of mass $m_0$ ("traveling observer") in a wormhole background, which is described by the reparametrization-invariant world-line action:

$$S_{\text{particle}} = \frac{1}{2}\int d\lambda \left[\frac{1}{e}\dot{x}^\mu \dot{x}^\nu G_{\mu\nu} - em_0^2\right]. \tag{45}$$

Using energy $\mathscr{E}$ and orbital momentum $\mathscr{J}$ conservation and introducing the *proper* world-line time $s$ ($\frac{ds}{d\lambda} = em_0$), the "mass-shell" constraint equation (the equation w.r.t. the "einbein" $e$) produced by the action (45)) yields:

$$\left(\frac{d\eta}{ds}\right)^2 + \mathscr{V}_{\text{eff}}(\eta) = \frac{\mathscr{E}^2}{m_0^2} \quad , \quad \mathscr{V}_{\text{eff}}(\eta) \equiv A(\eta)\left(1 + \frac{\mathscr{J}^2}{m_0^2 C(\eta)}\right) \tag{46}$$

where the metric coefficients $A(\eta), C(\eta)$ are those in (12).

Since the "effective potential" $\mathscr{V}_{\text{eff}}(\eta)$ in (46) is everywhere non-negative and vanishes only at the wormhole throat(s) ($\eta = \eta_0$, where $A(\eta_0) = 0$), "radially" moving test matter (*e.g.* a traveling observer) with zero "impact" parameter $\mathscr{J} = 0$ and with sufficiently large energy $\mathscr{E}$) will always cross from one "universe" to another within *finite* amount of its proper-time (see Fig.1). Moreover, this test matter (travelling observer) will "shuttle" between the turning points $\eta_\pm$:

$$\mathcal{V}_{\text{eff}}(\eta_\pm) = \frac{\mathcal{E}^2}{m_0^2} \quad , \quad \eta_+ > 0 \ , \ \eta_- < 0 \,, \tag{47}$$

so that in fact it will be trapped in the vicinity of the *LL-braneworld*. This effect is analogous to the gravitational trapping of matter near domain wall of a stable false vacuum bubble in cosmology [53].
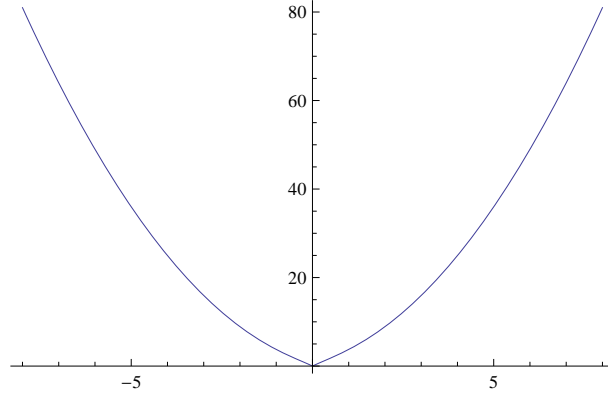


**Fig. 1** Shape of the "effective potential" $\mathcal{V}_{\text{eff}}(\eta) = A(\eta)$ with $A(\eta)$ as in (34). Travelling observer along the extra 5-th dimension will "shuttle" between the two 5-dimensional AdS "universes" crossing in either direction the 4-dimensional flat braneworld within *finite* proper-time intervals.

## 6 Discussion

Let us recapitulate the crucial properties of the dynamics of *LL-branes* interacting with gravity and bulk space-time gauge fields which enabled us to construct the *LL-braneworld* solutions presented above:

- (i) "Horizon straddling" – automatic positioning of *LL-branes* on (one of) the horizon(s) of the bulk space-time geometry.
- (ii) Intrinsic nature of the *LL-brane* tension as an additional *degree of freedom* unlike the case of standard Nambu-Goto *p*-branes; (where it is a given *ad hoc* constant), and which might in particular acquire negative values. Moreover, the variable tension feature significantly distinguishes *LL-brane* models from the previously proposed *tensionless p*-branes – the latter rather resemble *p*-dimensional continuous distributions of independent massless point-particles without cohesion among them.
- (iii) The stress-energy tensors of the *LL-branes* are systematically derived from the underlying *LL-brane* Lagrangian actions and provide the appropriate source

terms on the r.h.s. of Einstein equations to enable the existence of consistent non-trivial wormhole-like solutions.

- (iv) Electrically charged *LL-branes* naturally produce *asymmetric* wormholes with the *LL-branes* themselves materializing the wormhole "throats" and uniquely determining the pertinent wormhole parameters.
- (v) *LL-branes* naturally couple to Kalb-Ramond bulk space-time gauge fields which results in *dynamical* generation of space-time varying cosmological constant. (vi) *LL-branes* naturally produce *lightlike* braneworlds (extra dimensions are undetectable for observers confined on the *LL-brane* universe).

In our previous works we have also shown that:

- (vii) *LL-branes* trigger sequences of spontaneous compactification/decompactification transitions of space-time [18, 19, 20].
- (viii) *LL-branes* remove physical singularities of black holes [15].

The crucial importance of *LL-branes* in wormhole physics is underscored by the role they are playing in the self-consistent contsruction of the famous Einstein-Rosen "bridge" wormhole in its *original* formulation [55] – historically the first explicit wormhole solution. To this end let us make the following important remark. In several standard textbooks, *e.g.* [52, 56], the formulation of the Einstein-Rosen "bridge" uses the Kruskal-Szekeres manifold, where the Einstein-Rosen "bridge" geometry becomes *dynamical* (see ref.[52], p.839, Fig. 31.6, and ref. [56], p.228, Fig. 5.15). The latter notion of the Einstein-Rosen "bridge" is *not* equivalent to the original Einstein-Rosen's formulation in the classic paper [55], where the space-time manifold is *static* spherically symmetric consisting of two identical copies of the outer Schwarzschild space-time region ($r > 2m$) glued together along the horizon at $r = 2m$. Namely, the two regions in Kruskal-Szekeres space-time corresponding to the outer Schwarzschild space-time region ($r > 2m$) and labeled ($I$) and ($III$) in ref.[52] are generally *disconnected* and share only a two-sphere (the angular part) as a common border ($U = 0, V = 0$ in Kruskal-Szekeres coordinates), whereas in the original Einstein-Rosen "bridge" construction [55] the boundary between the two identical copies of the outer Schwarzschild space-time region ($r > 2m$) is their common horizon ($r = 2m$) – a three-dimensional *lightlike* hypersurface. In refs.[14, 17] it has been shown that the Einstein-Rosen "bridge" in its original formulation [55] naturally arises as the simplest particular case of static spherically symmetric wormhole solutions produced by *LL-branes* as gravitational sources, where the two identical "universes" with Schwarzschild outer-region geometry are glued together by a *LL-brane* occupying their common horizon – the wormhole "throat". An understanding of this picture within the framework of Kruskal-Szekeres manifold was subsequently given in ref.[57], which uses Rindler's identification of antipodal future event horizons.

One of the most interesting physical phenomena in wormhole physics is the well-known Misner-Wheeler "charge without charge" effect [54]. Namely, Misner and Wheeler have shown that wormholes connecting two asymptotically flat space-times provide the possibility of existence of electromagnetically non-trivial

solutions, where *without being produced by any charge source* the flux of the electric field flows from one universe to the other, thus giving the impression of being positively charged in one universe and negatively charged in the other universe.

In our recent paper [59] we found an opposite "charge-hiding" effect in wormhole physics, namely, that a genuinely charged matter source of gravity and electromagnetism may appear *electrically neutral* to an external observer. This phenomenon takes place when coupling self-consistently an electrically charged *LL-brane* to gravity and a *non-standard* form of nonlinear electrodynamics, whose Lagrangian contains a square-root of the ordinary Maxwell term:

$$L(F^2) = -\frac{1}{4}F^2 - \frac{f}{2}\sqrt{-F^2} \quad , \quad F^2 \equiv F_{\mu\nu}F^{\mu\nu} , \tag{48}$$

$f$ being a positive coupling constant. In flat space-time the theory (48) is known to produce a QCD-like effective potential between charged fermions [60]–[65]. When coupled to gravity it generates an effective global cosmological constant $\Lambda_{\rm eff} = 2\pi f^2$ as well a nontrivial constant radial vacuum electric field $f/\sqrt{2}$ [58]. When in addition gravity and nonlinear electrodynamics (48) also interact self-consistently with a charged *LL-brane* we found in ref.[59] a new type of wormhole solution which connects a non-compact "universe", comprising the exterior region of Schwarzschild-de Sitter black hole beyond the internal (Schwarzschild-type horizon), to a Levi-Civita-Bertotti-Robinson-type "tube-like" "universe" with two compactified dimensions (*cf.* [29, 30, 31]) via a wormhole "throat" occupied by the charged *LL-brane*. In this solution the whole electric flux produced by the charged *LL-brane* is pushed into the "tube-like" Levi-Civita-Bertotti-Robinson-type "universe" and thus the brane is detected as neutral by an observer in the Schwarzschild-de-Sitter "universe".

In the subsequent recent paper [66] we succeeded to find a truly "charge-confining" wormhole solution when the coupled system of gravity and non-standard nonlinear electrodynamikcs (48) are self-consistently interacting with *two* separate oppositely charged *LL-branes*. Namely, we found a self-consistent "two-throat" wormhole solution where the "left-most" and the "right-most" "universes" are two identical copies of the exterior region of the electrically neutral Schwarzschild-de-Sitter black hole beyond the Schwarzschild horizon, whereas the "middle" "universe" is of generalized Levi-Civita-Bertotti-Robinson "tube-like" form with geometry $dS_2 \times S^2$ ($dS_2$ is the two-dimensional de Sitter space). It comprises the finite-size intermediate region of $dS_2$ between its two horizons. Both "throats" are occupied by the two oppositely charged *LL-branes* and the whole electric flux produced by the latter is confined entirely within the middle finite-size "tube-like" "universe".

One of the most important issues to be studied is the problem of stability of the wormhole(-like) solutions with *LL-branes* at their "troats", in particular, the above presented *LL-braneworld* solutions in anti-de Sitter bulk space-times. The "horizon-straddling" property (15) of *LL-brane* dynamics will impose severe restrictions on the impact of the perturbations of the bulk space-time geometry.

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